Eur. Phys. J. C 1, 649–655 (1998)

THE EUROPEAN
PHYSICAL JOURNAL C

The low energy effective Higgs potential of the minimal nonlinear supersymmetric SU(5)-model and its phenomenology

H. Franz, B.R. Kim, M. Weber

III. Physikalisches Institut A, RWTH Aachen, D-52056 Aachen, Germany

Received: 28 April 1997

Abstract. The low energy effective Higgs potential of the minimal nonlinear supersymmetric SU(5)-Model is derived. The physical Higgs particle spectra are the same as those of the linear supersymmetric model MSSM: two scalar bosons, a pseudo-scalar and a charged scalar. Mass relations are derived and compared to those of the MSSM. Production cross sections are calculated for LEP1, LEP2, LC-500, LC-1000 and LC-2000 and discussed how far this model might be tested at these colliders.

Introduction

Almost all supersymmetric extensions of the Standard Model studied so far are linear supersymmetric models. However supersymmetry can also be realized nonlinearly and the question whether supersymmetry is realized linearly or nonlinearly in nature is still open. The first example for a nonlinear realization of supersymmetry is the Akulov-Volkov field constructed already in 1972 [1]. The formalism for constructing nonlinear supersymmetric extensions of the Standard Model was developed by Samuel and Wess [2]. In global nonlinear supersymmetric models the only new particle is the Akulov-Volkov field, which is a Goldstone fermion, a Goldstino. A Goldstino has not been observed in experiments [3]. In local supersymmetric models the Goldstino can be gauged away; it is absorbed into the gravitino which becomes massive [2]. In the flat space limit the supergravity multiplet decouples from ordinary matter and the only reminiscence of supersymmetry manifests itself in the Higgs sector.

A minimal nonlinear supersymmetric Standard Model in curved space has been constructed by one of us [4]. A remarkable feature of this model is that one needs at least two Higgs doublets and one Higgs singlet. Thus the Higgs boson spectrum is the same as that of the *Next To The Minimal Supersymmetric Standard Model* (NMSSM), which is a linear supersymmetric model.

However the structure of the Higgs potential is different in the two models. The phenomenology of this nonlinear model was investigated and the differences between the two models were worked out: [5–7].

We showed that this model most probably can be tested conclusively at future e^+e^- Linear Colliders with 500, 1000 and 2000 GeV.

In the meantime we also constructed a minimal nonlinear supersymmetric SU(5) model [8]. It turned out that the Higgs sector of this model at low energies is determined by two Higgs doublets, resembling that of the linear minimal supersymmetric Standard Model MSSM. In this note we derive the low energy effective Higgs potential of this nonlinear supersymmetric SU(5) model, mass eigenstates and mass relations, compare them to those of the MSSM and discuss how to distinguish between the two models. We also investigate the phenomenology of the model for e^+e^- linear colliders.

The model

It was shown in [8] that the minimal set of Higgs fields for a nonlinear supersymmetric SU(5) model consists of a 24-plet, a 5-plet and a $\bar{5}$-plet of Higgs fields.

In the notation of [8] the superpotential of the Higgs fields is given by

$$\begin{aligned}\mathcal{P} &= \mathcal{P}_{[24]} + \mathcal{P}_{[24,5]}\\ \mathcal{P}_{[24]} &= \lambda_1 \mathrm{Tr}\left(\Phi^{[24]\,2}_{(0)}\Phi^{[24]}_{(2)}\right) + \lambda_1 m \mathrm{Tr}\left(\Phi^{[24]}_{(0)}\Phi^{[24]}_{(2)}\right)\\ \mathcal{P}_{[24,5]} &= \lambda_2 \Phi^{[5]}_{(0)\,x_5}\left(\Phi^{[24]\;x_5}_{(0)\;y_5} + 3m_2\delta^{x_5}_{y_5}\right)\Phi^{[5]\,y_5}_{(2)}\\ &\quad +\lambda_3 \Phi^{[5]}_{(2)\,x_5}\left(\Phi^{[24]\;x_5}_{(0)\;y_5} + 3m_3\delta^{x_5}_{y_5}\right)\Phi^{[5]\,y_5}_{(0)}\\ &\quad +\lambda_4 \Phi^{[5]}_{(0)\,x_5}\left(\Phi^{[24]\;x_5}_{(2)\;y_5}\right)\Phi^{[5]\,y_5}_{(0)} \end{aligned}\tag{1}$$

In the flat space limit and denoting the physical Higgs fields by $\mathcal{H}^{\{24\}}$, $\mathcal{H}^{\{5\}}$ and $\mathcal{H}^{\{5\}}$ the scalar Higgs potential is given by

$$V = V_{[24]} + V_{[24,5]}\tag{2}$$

$$\begin{aligned}V_{[24]} &= \frac{1}{4}g_5^2\ \mathrm{Tr}\left(\left[\mathcal{H}^{\{24\}+}, \mathcal{H}^{\{24\}}\right]\right)^2\\ &\quad +\lambda_1^2\ \mathrm{Tr}\left(\mathcal{H}^{\{24\}\,2}\mathcal{H}^{\{24\}+\,2}\right)\end{aligned}$$

$$
\begin{aligned}
&+\lambda_1^2 m^2 \ \mathrm{Tr}\left(\mathcal{H}^{\{24\}}\mathcal{H}^{\{24\}^+}\right) \\
&-\frac{1}{5}\lambda_1^2 \ \mathrm{Tr}\left(\mathcal{H}^{\{24\}^2}\right)\ \mathrm{Tr}\left(\mathcal{H}^{\{24\}^{+2}}\right) \\
&+\lambda_1^2 m \ \mathrm{Tr}\left(\mathcal{H}^{\{24\}^2}\mathcal{H}^{\{24\}^+}\right) \\
&+\lambda_1^2 m \ \mathrm{Tr}\left(\mathcal{H}^{\{24\}^{+2}}\mathcal{H}^{\{24\}}\right) \qquad (3)
\end{aligned}
$$

$$
\begin{aligned}
V_{[24,5]} = \frac{1}{10}g_5^2 &\left(2|\mathcal{H}^{\{\bar{5}\}}|^4 + 2|\mathcal{H}^{\{5\}}|^4\right. \\
&\left.-5|\mathcal{H}^{\{\bar{5}\}}\mathcal{H}^{\{5\}}|^2 + |\mathcal{H}^{\{5\}}|^2|\mathcal{H}^{\{\bar{5}\}}|^2\right) \\
&+\lambda_4^2|\mathcal{H}^{\{\bar{5}\}}|^2|\mathcal{H}^{\{5\}}|^2 - \frac{\lambda_4^2}{5}|\mathcal{H}^{\{\bar{5}\}}\mathcal{H}^{\{5\}}|^2 \\
&+\lambda_1\lambda_4\mathcal{H}^{\{5\}^+}\mathcal{H}^{\{24\}^2}\mathcal{H}^{\{\bar{5}\}^+} \\
&+\lambda_1\lambda_4\mathcal{H}^{\{\bar{5}\}}\mathcal{H}^{\{24\}^{+2}}\mathcal{H}^{\{5\}} \\
&+\lambda_1 m\lambda_4\mathcal{H}^{\{5\}^+}\mathcal{H}^{\{24\}}\mathcal{H}^{\{\bar{5}\}^+} \\
&+\lambda_1 m\lambda_4\mathcal{H}^{\{\bar{5}\}}\mathcal{H}^{\{24\}^+}\mathcal{H}^{\{5\}} \\
&-\frac{1}{5}\lambda_1\lambda_4 \ \ \mathrm{Tr}\left(\mathcal{H}^{\{24\}^{+2}}\right)\left(\mathcal{H}^{\{\bar{5}\}}\mathcal{H}^{\{5\}}\right) \\
&-\frac{1}{5}\lambda_1\lambda_4 \ \ \mathrm{Tr}\left(\mathcal{H}^{\{24\}^2}\right)\left(\mathcal{H}^{\{5\}^+}\mathcal{H}^{\{\bar{5}\}^+}\right) \\
&+\lambda_2^2\left(\mathcal{H}^{\{\bar{5}\}}\mathcal{H}^{\{24\}}\mathcal{H}^{\{24\}^+}\mathcal{H}^{\{\bar{5}\}^+} + 9m_2^2|\mathcal{H}^{\{\bar{5}\}}|^2\right. \\
&\quad+\left[3m_2\mathcal{H}^{\{\bar{5}\}}\mathcal{H}^{\{24\}}\mathcal{H}^{\{\bar{5}\}^+}\right. \\
&\quad\left.\left.+3m_2\mathcal{H}^{\{\bar{5}\}}\mathcal{H}^{\{24\}^+}\mathcal{H}^{\{\bar{5}\}^+}\right]\right) \\
&+\lambda_3^2\left(\mathcal{H}^{\{5\}^+}\mathcal{H}^{\{24\}^+}\mathcal{H}^{\{24\}}\mathcal{H}^{\{5\}} + 9m_3^2|\mathcal{H}^{\{5\}}|^2\right. \\
&\quad+\left[3m_3\mathcal{H}^{\{5\}^+}\mathcal{H}^{\{24\}}\mathcal{H}^{\{5\}}\right. \\
&\quad\left.\left.+3m_3\mathcal{H}^{\{5\}^+}\mathcal{H}^{\{24\}^+}\mathcal{H}^{\{5\}}\right]\right) \qquad (4)
\end{aligned}
$$

The spontaneous symmetry breaking

$$\mathrm{SU}(5) \to \mathrm{SU}(3)\times\mathrm{SU}(2)\times\mathrm{U}(1)$$

can be achieved by the following vacuum expectation values:

$$
\left\langle\mathcal{H}^{\{24\}}\right\rangle_0 = v_H \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} \qquad (5)
$$

$$\left\langle\mathcal{H}^{\{5\}}\right\rangle_0 = 0$$

$$\left\langle\mathcal{H}^{\{\bar{5}\}}\right\rangle_0 = 0$$

with the extremum condition $v_H = m$.

As in a conventional SU(5) model v_H must be of order of GUT scale. Under this symmetry breaking all gauge bosons and adjoint Higgs bosons obtain a mass of order v_H except for the gauge bosons corresponding to SU(3) $\times$SU(2) $\times$ U(1) and the Goldstone bosons. The mass term of the quintuplet Higgs bosons $\mathcal{H}^{\{5\}}$ and $\mathcal{H}^{\{\bar{5}\}}$ is given by:

$$
\begin{aligned}
V_{[5,\bar{5}]\mathrm{mass}} = \frac{1}{2}&\left(m_{\{3\}}^2|H^{\{3\}}|^2 + m_{\{\bar{3}\}}^2|H^{\{\bar{3}\}}|^2\right. \\
&\left.+ m_{\{2\},1}^2|H_1|^2 + m_{\{2\},2}^2|H_2|^2\right) \qquad (6)
\end{aligned}
$$

with

$$
\begin{aligned}
m_{\{3\}}^2 &= 2\lambda_3^2(2v_H + 3m_3)^2 \\
m_{\{\bar{3}\}}^2 &= 2\lambda_2^2(2v_H + 3m_2)^2 \\
m_{\{2\},11}^2 &= 18\lambda_2^2(v_H - m_2)^2 \\
m_{\{2\},12}^2 &= 18\lambda_3^2(v_H - m_3)^2
\end{aligned}
$$

where we have used the following notation

$$
\begin{aligned}
\mathcal{H}^{\{\bar{5}\}\,T} &= \begin{pmatrix} H^{\{\bar{3}\}} \\ \epsilon H_1 \end{pmatrix} \ H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \\
\mathcal{H}^{\{5\}} &= \begin{pmatrix} H^{\{3\}} \\ H_2 \end{pmatrix} \ H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \qquad (7)
\end{aligned}
$$

The mass scale of the colour triplets $H^{(3)}$ and $H^{(\bar{3})}$ must be of order of GUT scale to be consistent with the proton lifetime, which is automatically the case for $m_3 \geq 0$ and $m_2 \geq 0$.

As for the mass parameters $m_{(2),11}^2$ and $m_{(2),12}^2$ are concerned one can fine tune v_H, m_2 and m_3 in such a way that the differences $v_H - m_2$ and $v_H - m_3$ are of order of electroweak scale, which we will assume in the following.

We remark that the only motivation for fine-tuning is the requirement that our SU(5)-Model should contain Higgs-doublets of electroweak scale. This is the same as with the conventional SU(5)-Model. It is however remarkable that the possibility for fine-tuning is only given for the doublet part. Without fine-tuning the doublet particles would in general also acquire masses of order of GUT scale.

At the electroweak scale the heavy particles can be functionally integrated out and the effective low energy Higgs potential is given in terms of the doublet Higgs H^1 and H^2. Hence we obtain the low energy effective Higgs potential omitting terms suppressed by $[m_W/m_{\mathrm{GUT}}]^n$:

$$
\begin{aligned}
V = &\frac{1}{8}(g_1^2+g_2^2)\left(|H_1|^2 - |H_2|^2\right)^2 + \frac{1}{2}g_2^2|H_1^\dagger H_2|^2 \\
&+\lambda^2\left[|H_1|^2|H_2|^2 - \frac{1}{5}|H_1^T\epsilon H_2|^2\right] \\
&+\mu_1^2|H_1|^2 + \mu_2^2|H_2|^2 + \mu_3^2(H_1^T\epsilon H_2 + \text{ h.c.}) \qquad (8)
\end{aligned}
$$

where g_1, g_2 and g_3 are the coupling constants of U(1), SU(2) and SU(3) respectively and μ_1, μ_2, μ_3 are mass parameters of electroweak scale.

At GUT scale the following relations hold

$$
\begin{aligned}
\lambda &= \lambda_4 \\
\mu_1^2 &= 9\lambda_2^2(v_H - m_2)^2 \\
\mu_2^2 &= 9\lambda_3^2(v_H - m_3)^2 \\
\mu_3^2 &= 3\lambda_1\lambda_4 v_H(m - v_H) \qquad (9)
\end{aligned}
$$

These relations are tree level relations and subject to change due to radiative corrections.

This is our model which will be investigated in the following. It contains two Higgs doublets. Therefore its Higgs particle spectrum is the same as that of the MSSM, but the structure of the potential is different.

Mass spectra

The spontaneous symmetry breaking

$$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \rightarrow \mathrm{SU}(3) \times \mathrm{U}(1)$$

occurs via the vacuum expectation values of the two Higgs doublets

$$\langle H_1 \rangle_0 = v_1/\sqrt{2}, \;\; \langle H_2 \rangle_0 = v_2/\sqrt{2}$$

With the notations

$$\begin{aligned} H_1 &= \begin{pmatrix} \frac{1}{\sqrt{2}}(v_1 + \eta_1 + ia_1) \\ h_1^- \end{pmatrix} \\ H_2 &= \begin{pmatrix} h_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \eta_2 + ia_2) \end{pmatrix} \end{aligned} \tag{10}$$

the extremum conditions yield

$$\begin{aligned} \mu_1^2 &= -\frac{1}{8} g^2 (v_1^2 - v_2^2) + \mu^2 \frac{v_2^2}{v^2} \\ \mu_2^2 &= \frac{1}{8} g^2 (v_1^2 - v_2^2) + \mu^2 \frac{v_1^2}{v^2} \\ \mu_3^2 &= -(\mu^2 + \frac{2}{5}\lambda^2 v^2) \frac{v_1 v_2}{v^2} \end{aligned} \tag{11}$$

and

$$\mu^2 = (\mu_1^2 + \mu_2^2) \geq \frac{1}{2} m_Z^2 (\tan^2\beta - 1)$$

for $\tan\beta = v_2/v_1 \geq 1$.

There are two physical scalar Higgs bosons S_1, S_2, one pseudo-scalar Higgs boson P and a charged one h^+

$$\begin{aligned} S_1 &= \cos\alpha\, \eta_1 + \sin\alpha\, \eta_2 \\ S_2 &= -\sin\alpha\, \eta_1 + \cos\alpha\, \eta_2 \\ P &= \sin\beta\, a_1 + \cos\beta\, a_2 \\ h^+ &= \sin\beta\, H_1^{-\,*} + \cos\beta\, H_2^+ \end{aligned} \tag{12}$$

with the mixing angles

$$\begin{aligned} \sin 2\alpha &= \frac{\sin 2\beta (m_Z^2 - m_P^2 + 2\mu^2)}{\sqrt{(m_P^2 - m_Z^2)^2 + 4\mu^2 \sin^2 2\beta\, (m_Z^2 - m_P^2 + \mu^2)}} \\ \cos 2\alpha &= \frac{\cos 2\beta (m_P^2 - m_Z^2)}{\sqrt{(m_P^2 - m_Z^2)^2 + 4\mu^2 \sin^2 2\beta\, (m_Z^2 - m_P^2 + \mu^2)}} \\ \tan 2\alpha &= \tan 2\beta \left(\frac{2\mu^2}{m_P^2 - m_Z^2} - 1 \right), \;\text{ if } m_P^2 \neq m_Z^2 \\ \sin 2\alpha &= 1, \;\text{ if } m_P^2 = m_Z^2 \end{aligned} \tag{13}$$

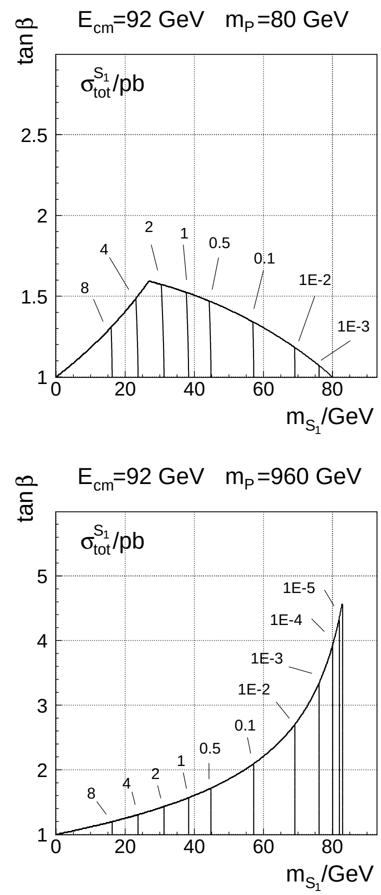

Fig. 1a,b. The contour plot of σ_1 in $\tan\beta - m_{S_1}$ plane at $m_P = 80$ GeV (960 GeV) for $\sqrt{s} = 92$ GeV

The masses m_{S_1}, m_{S_2}, m_P, m_C of the particles S_1, S_2, P,h^+ are given by

$$\begin{aligned} m_{S_1,S_2}^2 &= \frac{1}{2} m_Z^2 + \frac{1}{2} m_P^2 \mp \frac{1}{2} \left[(m_P^2 - m_Z^2)^2 \right. \\ &\qquad \left. + 4\mu^2 \sin^2 2\beta\, (m_Z^2 - m_P^2 + \mu^2) \right]^{\frac{1}{2}} \\ m_P^2 &= \frac{2}{5} \lambda^2 v^2 + \mu^2 \\ m_C^2 &= \frac{1}{4} g_2^2 v^2 + \frac{1}{2} \lambda^2 v^2 + \mu^2 \\ &= m_W^2 + \frac{1}{2} \lambda^2 v^2 + \mu^2 \end{aligned} \tag{14}$$

Now we give mass relations which may be used to characterize our model.

(a) $m_{S_1}^2 \leq m_Z^2 \left(\cos^2 2\beta + \frac{8}{5} \frac{\lambda^2}{g^2} \sin^2 2\beta \right)$

(b) $m_C^2 = m_W^2 + m_P^2 + \frac{2}{5} \frac{\lambda^2}{g^2} m_Z^2$

(c) $m_{S_1}^2 + m_{S_2}^2 = m_Z^2 + m_P^2$

(d) $m_{S_1}^2 * m_{S_2}^2 = m_Z^2 m_P^2 \cos^2 2\beta + \frac{8}{5} \frac{\lambda^2}{g^2} m_Z^2 \sin^2 2\beta$

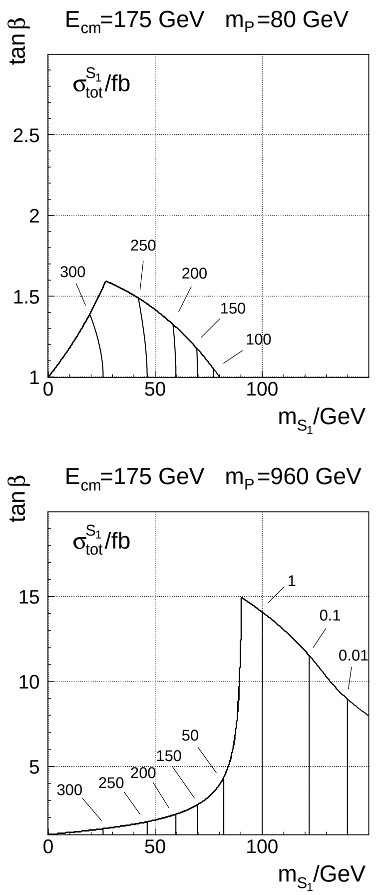

Fig. 2a,b. The contour plot of σ_1 in $\tan\beta - m_{S_1}$ plane at $m_P = 80$ GeV (960 GeV) for $\sqrt{s} = 175$ GeV

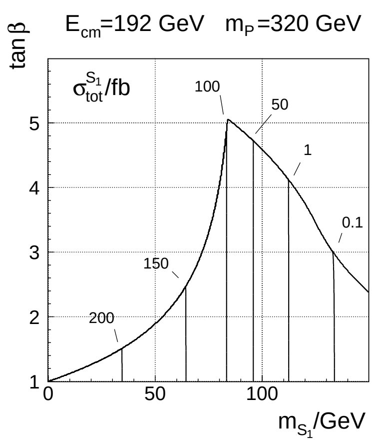

Fig. 3. The contour plot of σ_1 in $\tan\beta - m_{S_1}$ plane at $m_P = 320$ GeV for $\sqrt{s} = 192$ GeV

$$* \left[m_P^2 + m_Z^2 \left(1 - \frac{8}{5} \frac{\lambda^2}{g^2} \right) \right] \tag{15}$$

with $g^2 = g_1^2 + g_2^2$.

For comparison we give the corresponding relations of the MSSM:

$$\text{(a)} \;\; m_{S_1}^2 \leq m_Z^2 \cos^2 2\beta$$
$$\text{(b)} \;\; m_C^2 = m_W^2 + m_P^2$$

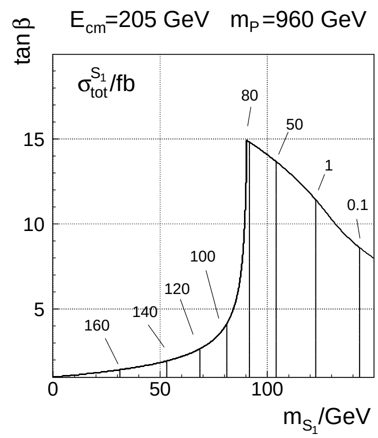

Fig. 4. The contour plot of σ_1 in $\tan\beta - m_{S_1}$ plane at $m_P = 960$ GeV for $\sqrt{s} = 205$ GeV

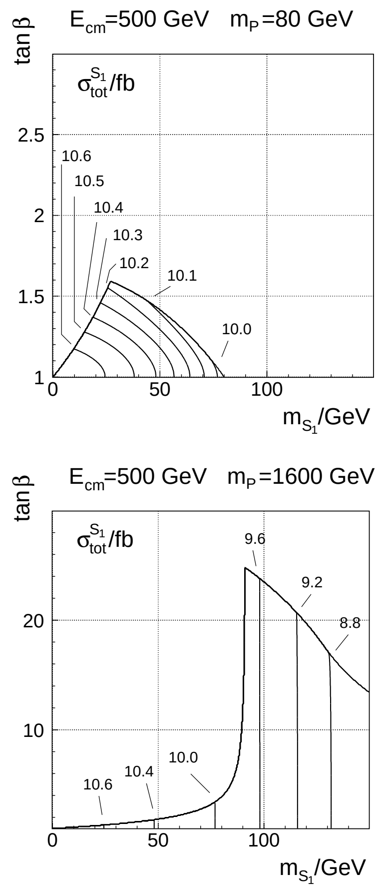

Fig. 5a,b. The contour plot of σ_1 in $\tan\beta - m_{S_1}$ plane at $m_P = 80$ GeV (1600 GeV) for $\sqrt{s} = 500$ GeV

$$\text{(c)} \;\; m_{S_1}^2 + m_{S_2}^2 = m_Z^2 + m_P^2$$
$$\text{(d)} \;\; m_{S_1}^2 * m_{S_2}^2 = m_Z^2 m_P^2 \cos^2 2\beta \tag{16}$$

The upper bound of m_{S_1}, (15a), contains the quartic coupling constant λ as in the nonlinear standard model [4] and NMSSM [9]. This is a remarkable result which implies that at least one Higgs boson has a mass of order m_Z. In case of $\lambda^2 \geq \frac{5}{8} g^2$ the upper bound of m_{S_1} is determined by the upper bound of λ and given by

$$m_{S_1} \lesssim \sqrt{\frac{5}{8}}\frac{\lambda}{g} m_Z \tag{17}$$

For $\lambda \approx 1$ and $g(m_Z) \approx 0.741$ the upper bound for m_{S_1} would be around $m_{S_1} \lesssim 157$ GeV.

The upper bound on λ can be determined through RGE-analysis by demanding that λ does not develop Landau poles up to GUT scale.

For the masses m_{S_2}, m_P and m_C one can not derive any theoretical upper bounds of the same quality as that of m_{S_1}. So in phenomenological analyses one should consider them up to the "unitarity bounds" of about 1000 $\sim$ 2000 GeV. We give some lower bounds which turn out to be useful in investigating particle production reactions:

$$\begin{aligned} m_P^2 &\geq \frac{1}{2} m_Z^2 \left(\tan^2\beta - 1\right) \quad \text{for } \tan\beta \geq 1 \\ m_C^2 &\geq m_W^2 + \frac{1}{2} m_Z^2 \left(\tan^2\beta - 1\right) \\ \left(m_Z^2 + m_P^2\right) &\geq m_{S_2}^2 \geq \frac{1}{2}\left(m_Z^2 + m_P^2\right) \end{aligned} \tag{18}$$

Production of Higgs bosons at e^+e^- colliders

In this section we investigate the production of Higgs bosons at LEP1, LEP2, LC-500, 1000 and 2000 GeV. The relevant reactions are

$$\begin{aligned} &\text{(i)} \quad e^+e^- \to Z \to ZS_i \to \bar{f}fS_i \\ &\text{(ii)} \quad e^+e^- \to Z \to \bar{f}f \to \bar{f}fS_i \\ &\text{(iii)} \quad e^+e^- \to Z \to PS_i \to \bar{f}fS_i \\ &\text{(iv)} \quad e^+e^- \to \gamma \to \bar{f}f \to \bar{f}fS_i \end{aligned} \tag{19}$$

The relevant Yukawa couplings of Higgs particles to up and down type quarks are given by

$$\begin{aligned} \mathcal{L}_{ZZS} &= \frac{g_2 m_Z}{2\cos\Theta_W} Z_\mu Z^\mu \left(\cos(\beta-\alpha)S_1 \right. \\ &\quad \left. + \sin(\beta-\alpha)S_2\right) \\ \mathcal{L}_{ZPS} &= \frac{g}{2} Z_\mu \left(-\sin(\beta-\alpha) P \overset{\leftrightarrow}{\partial^\mu} S_1 \right. \\ &\quad \left. + \cos(\beta-\alpha) P \overset{\leftrightarrow}{\partial^\mu} S_2\right) \\ \mathcal{L}_{\bar{f}fS} &= -\frac{g}{2m_Z} S_1 \left(m_d \frac{\cos\alpha}{\cos\beta}\bar{d}d + m_u \frac{\sin\alpha}{\sin\beta}\bar{u}u\right) \\ &\quad - \frac{2}{2m_Z} S_2 \left(m_d \frac{-\sin\alpha}{\cos\beta}\bar{d}d + m_u \frac{\cos\alpha}{\sin\beta}\bar{u}u\right) \\ \mathcal{L}_{\bar{f}fP} &= \mathrm{i}\frac{g}{2m_Z} m_d \tan\beta P \bar{d}\gamma_5 d \\ &\quad + \mathrm{i}\frac{g}{2m_Z} m_u \frac{1}{\tan\beta} P \bar{u}\gamma_5 u \end{aligned} \tag{20}$$

The dominant contributions come from the b-quark, f=b, so that in our analysis we shall concentrate on the production of b-quarks, assuming $m_b = 4.3$ GeV.

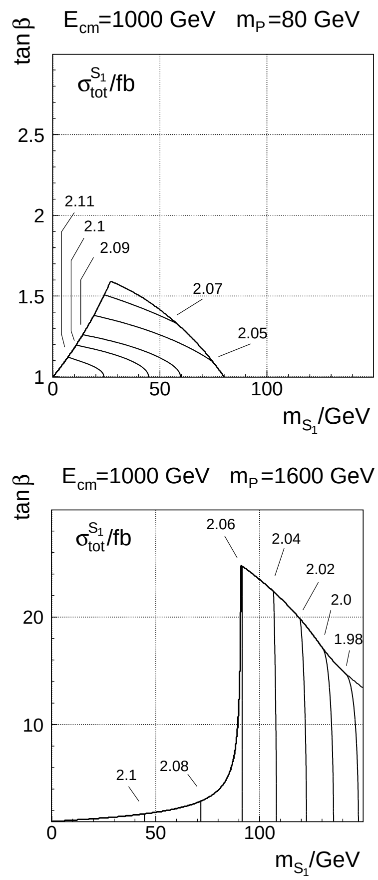

Fig. 6a,b. The contour plot of σ_1 in $\tan\beta - m_{S_1}$ plane at $m_P = 80$ GeV (1600 GeV) for $\sqrt{s} = 1000$ GeV

The production channel (iv) becomes comparable to channel (ii) at LC-500, 1000, 2000. The data from LEP1 yield an experimental lower bound of about 60 GeV on the Higgs boson mass of the standard model and about 44 GeV for the lightest scalar Higgs boson of MSSM. In the case of both NMSSM and the nonlinear supersymmetric standard model the LEP1 data do not exclude the existence of a massless scalar Higgs boson [6,7,10].

First we analyze the LEP1 data in the frame of the present model. Our model has three free parameters which can be taken as $\tan\beta$, m_{S_1} and m_P. As mentioned in the previous section no theoretical upper bound for m_P can be derived. So we systematically scanned the entire region $0 \leq m_P \lesssim 2000$ GeV. Figure 1a,b show the contour plot of σ_1, the production cross section of the lightest scalar Higgs boson S_1, in the the $\tan\beta - m_{S_1}$ plane for $m_P = 80$ GeV (960 GeV) at the center of mass energy $\sqrt{s} = 92$ GeV. The general trend is that σ_1 decreases with increasing m_{S_1}. They show that the region of $0 \leq m_{S_1} \leq 38$ Gev is excluded for a discovery limit of 1 pb. This lower bound remains for the entire region $0 \leq m_P \leq 2000$ GeV. Thus the LEP1 data exclude a Higgs boson mass m_{S_1} smaller than 38 GeV in the present model.

$$m_{S_1\,\mathrm{exp}} \geq 38 \text{ GeV} \tag{21}$$

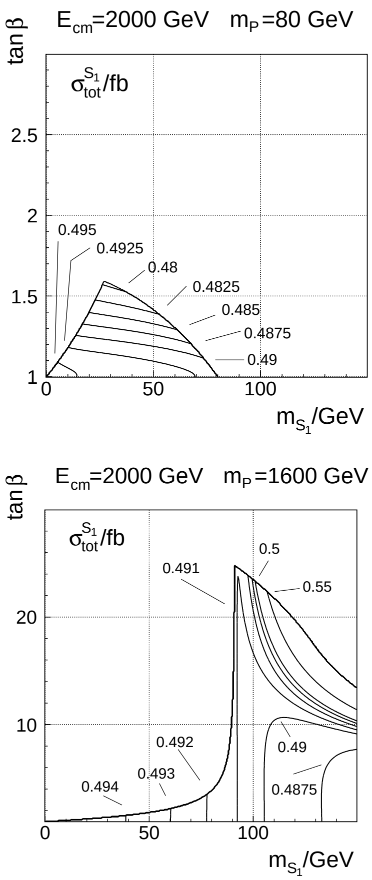

Fig. 7a,b. The contour plot of σ_1 in $\tan\beta - m_{S_1}$ plane at $m_P = 80$ GeV (1600 GeV) for $\sqrt{s} = 2000$ GeV

We have done the same analysis for LEP2. Figure 2a,b show the contour plots of σ_1 for $m_P = 80$ GeV (960 GeV) at $\sqrt{s} = 175$ GeV, Fig. 3 the plot for $m_P = 320$ GeV at $\sqrt{s} = 192$ GeV and Fig. 4 the plot for $m_P = 320$ GeV at $\sqrt{s} = 205$ GeV. Assuming a discovery limit of 50 fb we obtain the following lower bounds on m_{S_1}

$$
\begin{aligned}
m_{S_1} &\geq 82 \text{ GeV} \quad \text{for } \sqrt{s} = 175 \text{ GeV} \\
m_{S_1} &\geq 95 \text{ GeV} \quad \text{for } \sqrt{s} = 192 \text{ GeV} \\
m_{S_1} &\geq 100 \text{ GeV} \quad \text{for } \sqrt{s} = 205 \text{ GeV}
\end{aligned}
\tag{22}
$$

Now we come to LC-500, 1000, 2000.

With a theoretical upper bound for m_{S_1} of about 150 GeV σ_1 never vanishes in the entire parameter space at 500, 1000, 2000 GeV. Figures 5, 6 and 7 show the contour plots of σ_1 at m_P=80 GeV and 1600 GeV respectively. Our systematic analysis yields the following minima and maxima for σ_1

$$
\begin{aligned}
8.8 \text{ fb} \leq \sigma_1 \leq 10.7 \text{ fb} &\quad \text{for } \sqrt{s} = 500 \text{ GeV} \\
1.98 \text{ fb} \leq \sigma_1 \leq 2.12 \text{ fb} &\quad \text{for } \sqrt{s} = 1000 \text{ GeV} \\
0.48 \text{ fb} \leq \sigma_1 \leq 0.55 \text{ fb} &\quad \text{for } \sqrt{s} = 2000 \text{ GeV}
\end{aligned}
\tag{23}
$$

Now we come to the production of S_2. LEP1 and LEP2 are not capable to investigate S_2 due to negligible small production rates.

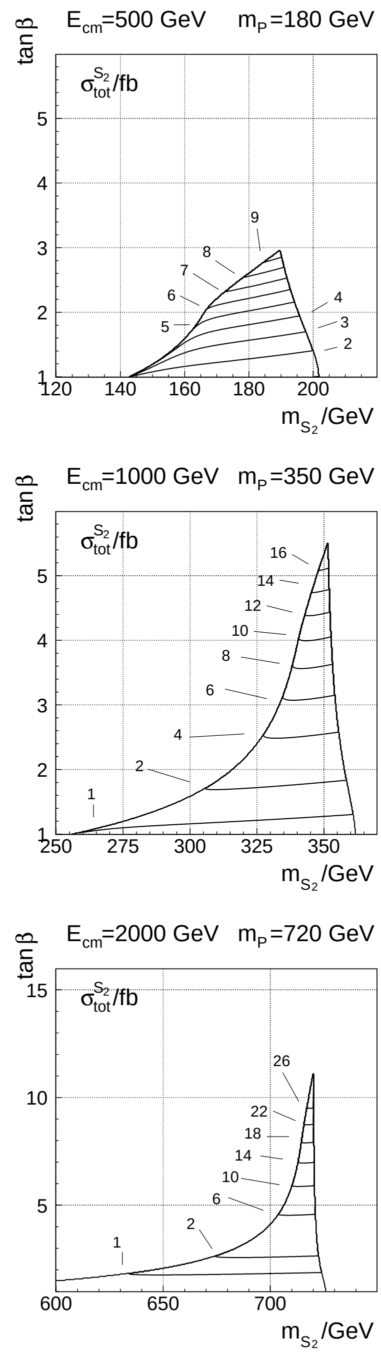

Fig. 8a–c. The contour plot of σ_2 in $\tan\beta - m_{S_2}$ plane for $\sqrt{s} = 500$ (1000, 2000) GeV

With respect to LC-500, 1000, 2000 we have found parameter regions where σ_2, the production cross section for S_2, is maximal.

For $\sqrt{s} = 500$ GeV σ_2 becomes as large as 9 fb in the region around $m_P \approx 180$ GeV, $m_{S_2} \approx 190$ GeV and $\tan\beta \approx 2.9$ as shown in Fig. 8a. For $\sqrt{s} = 1000$ GeV the interesting region is around $m_P \approx 350$ GeV, $m_{S_2} \approx 360$ GeV and $\tan\beta \approx 5$, where σ_2 is about 16 fb, whereas for $\sqrt{s} = 2000$ GeV $\sigma_{2\ \max}$ is about 26 fb in the region of

$m_P \approx 720$ GeV, $m_{S_2} \approx 700$ GeV and $\tan\beta \approx 10$. These regions are shown in Figs. 8b and 8c.

As for σ_P, the production cross section for the pseudoscalar Higgs boson P, the behaviour is the same as that of σ_2 because the main contributions come from the associated production, channel (iii) in (19). So the Figs. 8a–8c can be viewed as regions where σ_P reaches its maximum value of about the same magnitude as σ_2.

Conclusion

In this article we have investigated the low energy effective Higgs potential of the minimal nonlinear supersymmetric SU(5) model. It contains two Higgs doublet and thus constitutes a nonlinear supersymmetric alternative to the Higgs sector of the linear supersymmetric model MSSM.

We derived parameter independent mass relations and compared them to those of MSSM in order to be able to distinguish between the two models. Production cross sections at LEP1, LEP2, LC-500, LC-1000 and LC-2000 were calculated. It was shown that the existing LEP1 data may set a lower bound $m_{S_1} \gtrsim 38$ GeV, whereas LEP2 might push this lower bound beyond 82 GeV.

LC-500 would need a discovery limit smaller than 8.8 fb in order to test the model conclusively, whereas LC-1000 (2000) would require a discovery limit smaller than 1.98 fb (0.48 fb) for that purpose.

As for the second Higgs boson S_2 and the pseudoscalar P the situation is more difficult. At LEP1 and LEP2 the production rates are negligible small in the entire parameter space.

At higher cm energies the cross sections become relevant in a small region of the parameter space. At LC-500 the cross sections can be as large as 9 fb in the region around $m_P \approx 180$ GeV, $m_{S_2} \approx 190$ GeV and $\tan\beta \approx 2.9$. At LC-1000 (2000) the maximum cross sections occur in the region $m_P \approx 350$ GeV, $m_{S_2} \approx 360$ GeV and $\tan\beta \approx 5$ ($m_P \approx 720$ GeV, $m_{S_2} \approx 700$ GeV and $\tan\beta \approx 10$) and are about 16 fb (26 fb) for both S_2 and P. In these regions the production rate for S_2 is roughly equal to that for P because the main contributions come from the associated production, (19) channel (iii).

References

1. V.P. Akulov and D.V. Volkov: Phys. Lett. B46(1973)109
2. S. Samuel and J. Wess: Nucl. Phys. B221(1983)153; B226(1983)289; B233(1984)488
3. O. Nachtmann and M. Wirbel: Z. Phys. C23(1984)85; Z. Phys. C23(1984)199;
J.P. Ma, O. Nachtmann and M. Wirbel: Z. Phys. C30(1986)407;
O. Nachtmann and T. Schucker: Z. Phys. C39(1986)291
4. B.R. Kim: Z. Phys. C67(1995)337
5. H. Genten, S.W. Ham, B.R. Kim, S.K. Oh; Proceedings of the Workshop on Physics at LEP2, 1995, CERN96-01(1996)354; Proceedings of the Workshop on Physics and Experiments with Linear e^+e^- Colliders, Morioka-Appi, Iwate, Japan 8-12 Sept. 1995, (1996)566; Proceedings of the Workshop-Annecy, Gran Sasso, Hamburg, Feb. 1995 to Sept. 1995, DESY 96-123D(1996)429
6. S.W. Ham, H. Genten, B.R. Kim, S.K. Oh: Phys. Lett. B383(1996)178
7. H. Genten, S.W. Ham, B.R. Kim, S.K. Oh: to be published in Z. Phys. C (1997)
8. P. John, B.R. Kim: Z. Phys. C73(1996)169
9. J. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski, F. Zwirner: Phys. Rev. D39(1989)844;
M. Drees, Int. I. Mod. Phys. A4(1989)3635;
J.R. Espinoza and M. Quiros, Phys. Lett. B279(1992)92;
G.L. Kane, C. Kolda and J.D. Wells, Phys. Rev. Lett. 70(1993)2686;
S.F. King and P.L. White, SHEP-95-27, OUTP-95-31-P, to appear in Phys. Rev. D;
U. Ellwanger, M. Rausch de Traubenberg and C.A. Savoy, Phys. Lett. B315(1993)331; LPTHE-95-04, to appear in Z. Phys. C;
B.R. Kim, S.K. Oh, A. Stephan; Phys. Lett. B336(1994)200; Proceedings of e^+e^- Collisions at 500 GeV, The Physics Potential, DESY 92-123B(1992)697
10. B.R. Kim, S.K. Oh, A. Stephan; Proceedings of the Workshop on Physics and Experiments with Linear e^+e^- Colliders, Hawaii 26.-30. April 1993(1993)860